 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **STATISTICS**

SECOND SEMESTER – NOVEMBER 2012

# ST 2812 - TESTING STATISTICAL HYPOTHESES

Date : 03/11/2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

SECTION - A

Answer ALL questions. Each carries TWO marks: (10 x 2 = 20 marks)

1. Discuss the objective of Statistical Inference.

2. Explain Interval Estimation as an infinite decision problem with many possible correct

decisions.

3. Give the mathematical formulation of symmetry or invariance.

4. Describe minimax procedure.

5. Distinguish between size and level of significance of a test.

6. Show that one parameter exponential family possesses MLR property.

7. Define Similar test.

8. State Generalized Neyman-Pearson Theorem.

9. When do we say function is Maximal Invariant?

10. Define likelihood ratio test statistic and state the test criterion.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks: (5 x 8 = 40 marks)

11. Let β denote the power of a most powerful test of level α for testing simple hypothesis H

against simple alternative K. Prove that (i) β ≥ α (ii) α < β unless p0 = p1.

12. Let X1, … , Xn be a random sample from N(μ, σ2), where σ2 is known. Derive UMPT

of level α for testing H: μ ≤ μ0 versus K: μ > μ0.

13. Let ‘T’ denote the time required to get ‘r’ events in an inverse Poisson sampling with

process average rate λ. Show that the minimum number of events to be observed is

r = 14 in order to get a power greater than 0.8 at λ = 1.5 in a UMPT of level α = 0.05

for testing H: λ ≤ 0.5 versus K: λ > 0.5.

14. Obtain the UMPUT for H: λ = λ0 versus K: λ ≠ λ0 in the case of Poisson distribution

and deduce the ‘side conditions’ that are required to be satisfied.

15. If X ~ B(m, p1) and Y ~ B(n, p2) and are independent, then compare the two Binomial

populations using UMPUT for H: p1 ≤ p2 versus K: p1 > p2.

16. If the power function of every test function is continuous in θ, then show that any unbiased

test is similar on the boundary.

17. Define unbiased test and UMP test. Hence if there exists UMPT of level α for testing a

composite H against a composite K, then show that it is unbiased.

18. Explain locally most powerful unbiased level α test with an example.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. ( 2 x 20 = 40 marks)

19. Let X ~ Pθ with MLR in T(x). Establish the existence of UMPT for H: θ ≤ θ0 versus

K: θ > θ0 and obtain its form. β (θ) Show that its power β (θ) strictly increases for all θ

for which o < β (θ) < 1. Also show that the test minimizes β (θ) θ < θ0.

20. Let X1, … , Xn be a random sample from U(0, θ), θ > 0. For testing H: θ ≤ θ0 versus

K: θ > θ0, show that there exists more than one UMPT. In the same problem, obtain

UMPT for H: θ ≥ θ0 versus K: θ < θ0.

21. Let X1, … , Xn be a random sample from E(a, b) where both ‘a’ and ‘b’ are unknown.

Using the test statistic T = U / V, where U = X(1)  – a0 and V = ,

obtain the power function of the level α test

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for testing H: a = a0 versus KL: a < a0. Also obtain the power function of the level α test

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for testing H: a = a0 versus KR: a > a0.

22(a) Let X1, … , Xn be a random sample from N(μ, σ2), with both parameters unknown.

Derive the LRT of level α for testing H: σ2 = versus K: σ2 ≠ . (10)

(b) Consider the test for H: θ = θ0 versus K: θ ≠ θ0 based on a random sample of size ‘n’

from a distribution in the Cramer family. Derive the asymptotic null distribution of

the LRT statistic.

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